MATHEMATICS

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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

LINEAR PROGRAMMING

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. A general linear progamming problem can be stated as follows :

Given a set of m linear inequalities or equations in n variables, we wish to find non-negative value of theses variable which will satisfy these inequalities or equations and maximize or minimize some linear function of the variables.

The inequalities or equations are called the constaints and the function to be maximized or minimized is called the objective function whichcan be of maximization type of miminization type.

$Z = c_1 x_1 + c_2 x_2 + ... c_n x_n$...(i)

Subjected to

$$a_{11} x_1 + a_{12} x_2 + ... a_{1n} x_n \{\leq, =, \geq\} b_1$$
$$a_{21} x_1 + a_{22} x_2 + ... a_{2n} x_n \{\leq, =, \geq\} b_2$$
$$... \quad ... \quad ... \quad ... \quad ...$$
$$a_{m1} x_1 + a_{m2} x_2 + ... a_{mn} x_n \{\leq, =, \geq\} b_n$$

and, $x_1, x \quad y ... , x_n \geq 0$.

where,

(i) $x_1, x_2, ..., x_n$ are the variables whose value we wish to determine and are called the decision variables.

(ii) the linear function Z which is to be maximized or minimized is called the objective function.

(iii) the inequalities or equations in (ii) are called the constraints.

(iv) the set of inequalities in (iii) is known as the set of non-neganvity resptrictions.

(v) $b_i = (i = 1, 2, ..., m)$ respresent the requirement or availability of the ith constraint and the

column matrix $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$ is called teh requirement vector.

(vi) c_j (j = 1, 2, ..., n) represent the profit or cost to the objective function of the jth variable x_j and the row matrix $C = [c_1, c_2, ..., c_n]$ is called the profit (cost) matrix (vector).

(vii) the coefficients a_{ij} (i = 1, 2, ..., m; j = 1, 2, ..., n) are known as the technological or substituion coefficients.

(viii) the expression ($\leq, =, \geq$) means that one and only one of the signs $\leq, =, \geq$ holds for a particular constraint but the sign may vary from constraint to constraint.

EXERCISE-1

1. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

2. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machin B to produce a package of nuts while it takes 3 hours on macine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 2.50 per package of nuts and Rs. 1.00 per package of bolts. How many package of each should he produce each day so as to maximize his profit, if he operates his machines for at most 12 hours a day ? Formulate this mathematically and then solve it.

3. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minearls and 1400 of calories. Two foods A and B, are available at a cost of Rs. 4 and Rs. 3 per unit respectively. If one unit of A contains 200 units of vitamin, 1 unit of mineral and 40 calories and one unit of food B contains 100 units of vitamin, 2 units of minearls and 40 calories, find what combination of foods should be used to have the least cost ?

4. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. Rs. 4 per unit and F_2 costs Rs. 6 per unit one unit of food F_1 contains 2 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 5 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of maxture of these foods and also meets the minear nutritional requirements.

5. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kg and rice costs Rs. 4 per kg.

6. A factory munufactures two types of screws, A and B, each type requiring the use of two machines – an automatic and a hand-operated. It takes 4 minute on the automatic and 6 minutes on the hand-operated machines to manufacture a package of screws 'A', while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture of package of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacture can sell a package of screws 'A' at a profit of 70 P and scerws 'B' at a profit of Rs. 1. Assuming that that he can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit ? Determine the maximum profit.

7. A manufacturer makes two types A and B of tea-cups. Three machines are needed for the manufacture and the time a minutes required for each cup on the machines is given below :

	Machine		
	I	II	II
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each cup A is 75 paise and that on each cup B is 50 paise, show that 15 tea-cups of type A and 30 of type B should be manufactured in a day to get the maximum profit.

8. A factory owner purchase two types of machines, A and B, for his factory. The requirements and limitations for the machines are as follows :

	Area occupied by the machine	Labour force for each machine	Daily output in units
Machine A	1000 sq. m	12 men	60
Machine B	1200 sq. m	8 men	40

He has an area of 7600 sq. m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output ?

9. A company manufactures two types of novelty Souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for isembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is 50 paise each for type A and 60 paise each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit ?

10. A Manufacrurer makes two products *A* and B. Product *A* sells at Rs 200 each and takes 1/2 hour to make. Product *B* sells at Rs 300 each and takes 1 nour to make. There is a permanent order for 14 of product *A* and 16 of product B. A working week consists of 40 hours of production and weekly turnover must not be less than Rs 10000. If the profit on eacli of product *A* is Rs 20 and on product B is Rs 30, then how many of each should be produced so that the profit is maximum. Also, find the maximum profit.

11. A manufacturer produces two types of steel trunks. He has two machines *A* and B. For completing, the first type of the trunk requires 3 hours on machine *A* and 3 hours on machine *E,* whereas the second type of the trunk requires 3 hours on machine *A* and *2* hours on machine B. Machines *A* and B can work at most for 18 hours and 15 hours per day respectively. He earns *a* profit of Rs 30 and Rs 25 per trunk of the first type and the second type respectively. How many trunks of each type must the make each day to make maximum profit ?

12. A gardener has supply of fertilizer of type I which consists of 10% nitrogen and 6% phosphoric acid and type II fertilizer which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosporic acid for his crop. If the type I fertilizer costs 60 paise per kg and type H fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that nutrient requjremerts are met at a minimum cost. What is the minimum cost ?

13. A small firm manufacturers items *A* and B. The total number of items *A* and *B* that it can manufacture in a day is at the most 24. Item *A* takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item *A* be Rs 300 and one unit of item *B* be Rs 160, how matny of each type of item be produced to maximize the profit? Solve the problem graphically.

14. A company manufactures two types of toys *A* and B. Type *A* requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for

assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs 50 each on type *A* and Rs 60 each-eft type B. How many toys of each type should the company manufacture in adayto maximize the profit ?

15. A company manufactures two articles *A* and *B*. There are two departments through which these articles are processed : (i) assembly and (ii) finishing departments. The maximum capacity of the first department is 60 hours a week and that of other department is 48 hours per week. The product of each unit of article A requires 4 hours in assembly and 2 hours hi finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs 6 for each unit of *A* and Rs 8 fopeacn unit of *B,* find the number of units of *A* and B to be produced per week in order to have maximum profit.

16. Alifm makes items *A* and B and the total number of items it can make in a day is 24. It takes one hour to make an item of *A* and half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of *A* is Rs 300 and on one item of B is Rs 160. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

17. A manufacturer produces two types of steel trunks. He has two machines *A* and 6. rst types of the trunk requires 3 hours on machine *A* and 3 hours on machine "B. *Tne* second type of trunk requires 3 hours on machine *A* and 2 hours on machine *B.* Machines *A* and *B* are run daily for 18 hours and 15 hours respectively. There is a profit of Rs 30 on the first type of the trunk and Rs 25 on the second type of the trunk. How many trunks of each type should be produced and sold to make maximum profit ?

18. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring, and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

19. A library has to accomodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weigh 1 kg and $1\frac{1}{2}$ kg each respectively. The shelf.is 96 cm long and almost can support a weight of 21 kg. How should the shelf be filled with the books of two tyjaesirToFder to include the greatest number of books? Make it as an LPP and solve it graphically.

20 One kind of cake requires 300 gm of flour and 15 gm of fat, another kind of cake requires 150 grn of flour and 30 gm of fat. Find the maximum number of makes which can be made from 7.5 kg of flour and 600 gm of fat, assuming that there is no shortage of the other ingradients used in making the cakes. Make it as an LPP and solve it graphically.

21. Two godowns A and B, have grain storage capacity of 100 quintals and 50 qumteis respectively. They supply to 3 ration shops, D, E and F, whose requires are 60, 50 and 40 quintals respectively. The cost of transportation per quinel from the godowns to the shops are given in the following table:

	Transportation cost per quintal (in Rs.)	
From	A	B
To		
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum ?

22. An oil company has two depots, *A* and B, with capacities of 7000 litres and 4000 litres respectively. The company is to supply oil to three petrol pumps, D, E, F whose requirements are 4500, 3000 and 3500 litres respectively. The distance (in km) between the depots and petrol pumps is given in the following table :

	Distance (in Km)	
From	A	B
To		
D	7	3
E	6	4
F	3	2

Asssuming that the transportation cost per km is Rs. 1.00 per litre, how should the delivery be scheduled in order that the transportation cost is minimum ?